Multi-player games in which co-operation between players is allowed can lead to Big Trouble as any politician will confirm. The reader is therefore advised to take great care when opening

A PANDORA'S BOX OF NON-GAMES

Anatole Beck & David Fowler

In the study of Games, as in many other intellectual pursuits, one of the important problems is to Find the Question. When the question has been found, the answer may be sought in good time. In Game Theory, there are simple games, like the matrix games, and very, very complex games. Today, the centre of game theory is occupied by the theory of cooperative games, in which it is not yet known what an answer would be, much less how to find one. We will include below a few simple cooperative games. In addition, we will exhibit some things which are almost certainly games, except that they are so ephemeral, so indistinct, that they still defy analysis. Unlike Chess or Go, where the complexity arises from the multiplicity of possible strategies, the difficulty here arises exactly because of the great simplicity. No doubt, if the games were more complex, the difficulties would be hidden. Let's start with an English game:

1. Finchley Central.

Two players alternate naming the stations of the London Underground. First to say "Finchley Central" wins. It is clear that the "best" time to say Finchley Central is exactly before your opponent does. Failing that it is good that he should be considering it. You could, of course, say "Finchley Central" on your second turn. In that case, your opponent puffs on his cigarette and says "Well,..." Shame on you.

The next game is similar to F.C., except that it is played for money between people who have some.

2. Penny Pot.
Players alternate turns. At each turn, a player either adds a penny to the pot or takes the pot. Winning player makes first move in next game. Like F.C., this game defies analysis. There is, of course, the stable situation in which each player takes the pot, whenever it is not empty. This is a solution?

Penny Pot has an interesting variant,

3. Penny Pot with interest
The pot is a bank account, on which the players draw interest, which they share.

The next game is a three-person symmetric game:

4. Lucky Pierre.
Each of the three players chooses a positive integer. If all three numbers are different, then the one in the middle collects a franc from each of the others. If two are the same, then the odd man out collects. If all are the same, then no dice. This game has some interesting analysis. If two of the players gang up on the third, then they can take 4 and 5. No matter what happens, one of them wins. They share the loot. To make the game have meaning, there has to be some sort of real bar to collusion. If there is, and if all the players are thought of as intelligent (where did that hypothesis come from?), then we have the following chain of theorems:

Theorem 4.1: No one ever plays 1.
Proof: There is almost no hope of winning when you play 1. Only if the other two tie, can you get anything. And then not much. QED (?).

Theorem 4.2: No one ever plays 2.
Proof: Since no one ever plays 1, by the previous theorem, the same reasoning applies to 2. QED (??).

Theorem 4.3: No one ever plays 3.
Proof: Obvious (?????????)!

There are other theorems, too numerous to mention, and which imply

Theorem: No one ever plays.
Proof: Left to the reader as an exercise. (!!).

This analysis is similar to the analysis of the surprise test, for those of you who know it. Another game with the same analysis is

5. <u>Big Number</u>.

Two players. Each chooses a positive integer. The owner of the smaller integer pays a rupee to the owner of the larger. Theorems 5.1, 5.2, 5.3, <u>etc</u>. are left for formulation to the reader.

Big Number differs sharply from

6. <u>More Money</u>.

Two players with literally infinite resources. Each bets an amount of money. The larger bettor wins the stake of the smaller (see Marx, <u>passim</u>). Here, it is common to see players betting pennies, except from time to time. Similar to F.C. and Penny Pot, in some ways. The object, of course, is to win, rather than come out winning. Only when you have infinite resources can there be a distinction between these.

7. <u>Come to Dinner</u>.

Two players, Source and Sink. Mr. Source offers dinner to Mr. Sink ("Come to Dinner"). Mr. Sink refuses, indicating that he would like dinner, but courtesy forbids (e.g. "It is late, and your wife is not expecting me.") Source insists ("We have Stroganoff tonight, and Denise always makes plenty.") and Sink ducks again. Finally Source says, "Very well, some other time." Or Sink says, "All right, since you insist." Whoever says this line WINS. The game is played for two prizes, Dinner and Honour. The principal object is to get (resp. avoid giving) Dinner, and to do so while obtaining as much Honour (measured in rounds) as possible. Both players accrue Honour, but no amount of Honour can compensate for the loss of Dinner. The pay-off is not Archimedian. Note the similarity of this game to F.C. and P.P.

8. <u>Tweedledum and Tweedledee</u>.

The Red Queen offers 1000 marks to Tweedledum and Tweedledee if they will agree on how to share it; time limit. Tw...m offers 50-50, but Tw...e holds out for DM 650.

Tw...m: If you don't take this, you'll get nothing. Come on, its DM 500 or nothing.

Tw...e: Same goes the other way; do you want the DM350 or not?

If you think this is a simple game, imagine the inventor who has a device which can save the telephone company ⍭ 1000000 a year. The device is patented, and no one but the telephone company can use it. How much is his share of the take?

9. <u>Winken, Blinken, and Nod</u>.

Same Red Queen, having failed to dispose of her swag to Tw + Tw, Ltd., offers same to Wn, Bn, + N on the following terms. If they all will agree on the mode of sharing, they get the DM 1000. If two agree without the third, they get the following, depending on which two they are; odd man gets nothing: Wn+Bn 500, Wn+N 750, Bn+N 600. No agreements mean no dough. Who gets how much? Shame on you if you don't hit the patsy for the whole grand. Tw+Tw and Wn+Bn+N are called cooperative games. If you think these are screwy, you should see what happened the day the Red Queen offered some cash to Haupt, Voll, Blut, and Wunden.

10. An Infinite Game.

A real mathematician's game with a real mathematician's solution. A and B alternate choosing positive ($\neq 0$) real numbers to form a decreasing sequence; they play forever. At the Trump of Doom they add up their choices (infinitely many). If the sum is infinite or rational, A wins. Otherwise B. How does it come out? (Solution left to reader - see next issue for our version).